# An acoustic Casimir effect[†]

Andrés Larraza and Bruce Denardo

Department of Physics, Naval Postgraduate School, Monterey, CA  93943

Theoretical and experimental results are presented for the force law between two rigid, parallel plates due to the radiation pressure of band–limited acoustic noise. Excellent agreement is shown between theory and experiment.  While these results constitute an acoustic analog for the Casimir effect, an important difference is that band-limited noise can cause the force to be *attractive* or *repulsive* as a function of the distance of separation of the plates.  Applications of the acoustic Casimir effect to background noise transduction and non-resonant acoustic levitation are suggested.

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In the Casimir effect [1], two closely spaced uncharged parallel conducting plates mutually attract because their presence changes the mode structure of the quantum electromagnetic zero point field (ZPF) relative to free space. If the plates are a distance d apart, the force per unit area is $f = \pi^2 \hbar c/240\, d^4$, where $\hbar$ is the reduced Planck's constant and c is the speed of light in vacuum. The force can be calculated from the difference between the vacuum electromagnetic energy for infinite plate separation distance and a finite plate separation distance [1]. Lamoreaux [2] has recently provided conclusive experimental verification of the Casimir force.

The attraction between the two parallel plates can be understood in terms of the radiation pressure exerted by the plane waves that comprise the homogeneous, isotropic ZPF spectrum. In the space between the conducting plates, the modes formed by reflections off the plates act to push the plates apart. The modes outside the cavity formed by the plates act to push the plates together. The difference between the total outward pressure and the total inward pressure is the Casimir force per unit area. [3] Because the energy per mode of the zero point field has the same value $\frac{1}{2}\hbar\omega$ between and outside the plates, one may incorrectly be led to attribute the attractive character of the force as due to the fact that there are fewer modes between the plates. Surprisingly, as we show below, the force can be *repulsive* for band-limited noise.

Because the ZPF can be thought of as broadband noise of an infinite spectrum, it should be possible to use an acoustic broadband noise spectrum as an analog to at least some ZPF effects. An acoustic spectrum has several advantages. Because the speed of sound is six orders of magnitude less than the speed of light, the length and time scales are more manageable and measurable. Also, in an acoustic field, the shape of the spectrum as well as the field intensity can be controlled. In this letter, we report theory and measurements of the force between two rigid parallel plates in an externally-generated band-limited noise field. [4]

A simple calculation shows that for the same separation distance, the Casimir force due to the electromagnetic zero point is at least six orders of magnitude greater than the Casimir force due to zero point phonons, because the speed of light is six orders of magnitude greater than the speed of sound and because the phonon spectrum has a natural high frequency cutoff at the Debye temperature. To be distinguished from the effects of the zero point field, a Casimir-like effect due to thermal phonons and photons would require separation distances $d > hc/2kT$ which would make the force extremely difficult to measure. Driven electromagnetic white noise (composed of real photons) would yield forces much smaller than acoustically driven noise. Thus, the best choice for a Casimir analog is acoustically driven noise. The forces we measure in this case are the equivalent of 30 mg while the forces Lamoreaux measured due to actual Casimir effect are equivalent to 10 micrograms.

One of the key ideas in the derivation of the ZPF Casimir force is the fact that the energy per mode $\frac{1}{2}\hbar\omega$ is the same for modes both outside and between the plates, which can be understood with the adiabatic theorem. To this purpose, imagine that the plates are initially far apart so that the spectral intensity of the ZPF is that of free space. If we now adiabatically move the walls towards each other, the modes comprising the ZPF will remain in their ground state; only their frequencies will be shifted in such a way that the ratio of the energy per mode E to the frequency $\omega$ remains constant, or $E/\omega = \hbar/2$. Thus, the main effect of the boundaries is to redistribute ground state modes of which there is an infinite number.

In the acoustic Casimir effect, in contrast to the ZPF Casimir effect, broadband acoustic noise outside two parallel rigid plates drives the discrete modes between the plates. The adiabatic theorem does not apply in this case both because of inherent losses in the system and because the spectrum can be arbitrary. In general, when the response and drive amplitudes are expressed in the same units, the response is approximately the quality factor Q multiplied by the drive ("Q amplification"). The energy per mode being the same in the ZPF Casimir effect therefore implies that the space between the plates cannot be considered as a resonant cavity unless the quality factor of each mode is unity. While external drivers can provide a steady state noise spectrum from which we can infer the energy per mode by dividing by the density of states $\omega^2/2\pi^2c^3$, this energy may be different in the cavity formed by the plates as a result of Q amplification. However, for this open resonant cavity the quality factor is poor, so we may assume it to be equal to unity, which renders the energy per mode equal to its value in free space.

In general, the radiation pressure of a wave incident at angle $\theta$ on a rigid plate is

$$P = \frac{2I}{c}\cos^2\theta \ , \tag{1}$$

where $I$ is the average intensity of the incident wave and c is the wave speed. The factor of two is due to perfect reflectivity assumed for the plate. Eq. (1) follows from the time-averaged second-order acoustic pressure, which equals the time-averaged potential energy density minus the time-averaged kinetic energy density. [5] When the acoustic case is constrained to one dimension, mass conservation yields an explicit dependence of the radiation pressure on the elasticity of the medium characterized by $\gamma$, the ratio of specific heats, namely $P = (1+\gamma)I/c$. [6] However, for the three-dimensional open geometry in our case, the constraint due to mass conservation does not apply and the acoustic and electromagnetic expressions for the radiation pressure at a perfectly reflecting surface are identical. [7]

With appropriate filters, one may shape the spectrum of an acoustic driver and obtain, in principle, different force laws. For an isotropic noise spectrum with spectral intensity $I_\omega$, (measured by a microphone) the spectral intensity in the wavevector space of traveling waves is $I_{\mathbf{k}} = cI_\omega/4\pi k^2$, where the wavevector **k** has magnitude $k = \omega/c$. We choose the z axis to be normal to the plate, so that $k_z = k \cos\theta$. From Eq. (1), the total radiation pressure due to waves that strike the plate is then

$$P_{out} = \frac{2}{c}\int dk_x dk_y dk_z I_k \cos^2\theta \quad , \tag{2}$$

where the integration is over **k** values corresponding to waves that strike the plate.

Regarding the discrete modes between the plates, for convenience we continue to deal with the traveling wave modes. We label these modes with wavevector components $k_x = n_x\pi/L_x$, $k_y = n_y\pi/L_y$, and $k_z = n_z\pi/L_z$, where $n_x$, $n_y$, and $n_z$ are signed integers and $L_x$, $L_y$, and $L_z$ are the dimensions between the plates. As before, the z axis is chosen to be normal to the plates. As a result of the quality factor of the modes being approximately unity, the intensity $I_{in}(\mathbf{k})$ of each mode between the plates is expected to be approximately the same as the outside broadband intensity in a bandwidth equal to the wavevector spacing of the inside modes: $I_{in}(\mathbf{k}) = I_{\mathbf{k}}\Delta k_x\Delta k_y\Delta k_z$, where $\Delta k_i = \pi/L_i$. In the limit of large dimensions, this expression for the inside intensity yields the correct wavevector spectral intensity $I_{\mathbf{k}}$.

We assume that the dimensions $L_x$ and $L_y$ of the plates are sufficiently large that the corresponding components of the wavevectors are essentially continuous. Thus, in comparison to Eq. (2), the total inside pressure is

$$P_{in} = \frac{2}{c}\sum \Delta k_z \int dk_x dk_y I_{\mathbf{k}} \frac{k_z^2}{k^2} , \tag{3}$$

where $\Delta k_z = \pi/L_z$ and the sum is over values of $n_z > 0$.

The difference $P_{in}$–$P_{out}$ is the force f per unit area between the plates, which is a continuous and piecewise differentiable function of the separation distance between the plates. It can be shown (see below) that the force can alternate between negative (attractive force) and positive (repulsive force) values as the plate separation distance or the band-limiting frequencies are varied. On the other hand, if the lower frequency in the band is zero, the force is always attractive.

In an experiment dealing with an acoustic analog to the Casimir effect, an important question is whether other nonzero time-averaged (dc) effects can play an important role. The only second order dc effects in acoustics are radiation pressure and streaming. Any other dc effect would be fourth order in the acoustic pressure, and at least 40 dB smaller in our case. Employing smoke in the apparatus described below, we detected no acoustic streaming when driving with broadband acoustic noise at the intensity level used in the experiment. Because the noise in our experiment can be thought as collection of monochromatic waves over a band of frequencies with randomly varying phases, we would expect very little or no streaming when the characteristic time of phase variations is less that the diffusion time. Furthermore, because acoustic streaming is driven along the boundary from a pressure antinode to a pressure node, in the presence of broadband noise the pressure nodes and antinodes of the different noise components are densely distributed along the boundary, thus reducing or eliminating the streaming.

Two 15.00 cm diameter plates were used for the acoustic Casimir force measurement (Fig. 1). The bottom plate is 6.35 mm (1/4 inch) thick aluminum, attached to the top of a step motor mount. The top plate is 5.57 mm (7/32 inch) thick PVC which was vacuum-aluminized and hung beneath the analytical balance by an aluminum bar screwed into the plate and attached to the balance by a hook (Fig. 1). The weight of the top plate, 168.1653 g, is well within the 200 g maximum capacity of the balance, whose resolution is of 0.01 mg. Both plates were grounded to a common ground to eliminate electrostatic effects, thereby minimizing fluctuations in the force measurements. The acoustical chamber was made from a 1/4 in. thick steel propane tank. The amplified band-limited output of an analog noise source drives six compression drivers that provide the desired acoustic noise intensity within the acoustic chamber. The acoustic noise is nearly homogeneous and isotropic.

The step motor was mounted on a small aluminum optical bench with three-point leg adjustments, centered in the acoustic chamber 46 cm from the tank access. The number of steps (1 to 255 steps) and direction (up or down) are varied with a microchip controller. Attached to a machined screw with 20 threads/inch, the step motor mount yields a displacement ranging from 6.35 μm for one step (1.8 degrees) to 1.27 mm for 200 steps (360 degrees). We employed plate spacing increments of 1.27 mm through the full 6.3 cm range of the step motor mount.

The initial plate spacing was determined with a spark plug gap gauge and the plate separation distance was verified to be uniform by measurements at different locations between the plates. Once the balance reading locked in on the weight of the top plate, the balance was tared to zero and all force measurements were made relative to this zero. The acoustic noise field was turned on for each plate separation distance and the force measurement recorded once the balance readout first locked-in. The sound field was then turned off and the reading of the balance was verified to return to zero.

We selected a uniform broadband noise spectrum between roughly 5 and 15 kHz. The lower limit was selected well above the lower modes of the tank in order to excite a noise distribution as homogeneous as possible. The upper limit was due to the compression drivers rolling off 20 dB from 15 to 20 kHz. Figure 2 shows the measured noise spectrum which, except for 5 dB variations throughout, is nearly flat within the spectral range of 4.8 to 16 kHz. The total intensity is 133 dB (re $10^{-12}$ W/m$^2$).

Using a spark plug gauge, we measured the initial plate separation distance to be 0.76 mm. As shown in Fig. 3, throughout a range of distances less than the smallest half wavelength (no modes between the plates), the measured force is approximately independent of distance, and agrees with the expected value for an intensity level of 133 dB. When the half wavelength of the highest frequency fits between the plates (10.63 mm for 16 kHz), the force begins to decrease non-monotonically, and becomes repulsive (i.e., negative) at 30 – 40 mm. Based on the experimental spectral range of 4.8 to 16 kHz for the theory, with a total intensity of 133 dB, the curves are from the theory (2) and (3) with no adjustable parameters for a flat spectrum (solid curve) and a piecewise power–law spectrum that approximates the experimental spectrum.

The repulsive force can be understood as follows. When the distance between the plates is comparable to the half wavelength associated with the lower edge of the frequency band, the corresponding modes inside the plates have wavevectors that are nearly perpendicular to the plates. However, the modes outside the plates corresponding to the same frequencies are spread over all possible angles of incidence. Thus for the same total intensity, the momentum transfer due to waves inside the plates is over a narrow cone while the momentum transfer due to waves outside the plates extends over all angles, leading to a repulsive force.

Experimental evidence of attractive and repulsive forces within a finite acoustic bandwidth suggests new means of acoustic levitation. The force between two objects can be manipulated by changing the distance between the objects and/or varying the spectrum. While the Casimir force is small compared to the force of the Earth's gravity, in a low gravity environment a method of material control through the manipulation of an acoustic noise spectrum or plate geometry may be possible. The acoustic Casimir effect can also be a potential tool in noise transduction because a direct measurement of the force can determine the total intensity of background noise. The shape of the force over distance is effectively an instantaneous time average over all frequencies and may provide an alternative to measurements of background noise.

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[4] Further details regarding the background, theory, and measurement of this acoustical Casimir effect are in A. Larraza, C. D. Holmes, R. T. Susbilla, and B. Denardo, “The force between two parallel rigid plates due to the radiation pressure of broadband noise: An acoustic Casimir effect,” J. Acoust. Soc. Am. **103**, 2267-2272 (1998). The experiment was the culmination of two M.S. theses at the Naval Postgraduate School, Monterey, California: R. T. Susbilla, “Casmir Acoustics” (1996), and C. D. Holmes, “Acoustic Casimir Effect” (1997).

[5] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, (Pergamon Press, London 1959), Sec. 64.

[6] Lord Rayleigh, "On the pressure of vibrations," Philos. Mag. **3**, 338-346 (1902). Lord Rayleigh, "On the momentum and pressure of gaseous vibrations, and the connection with the virial theorem," Philos. Mag. **10**, 364-374 (1905).

[7] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, (Pergamon Press, London 1959), Sec. 65, considered a plane wave incident at an arbitrary angle at the interface of two fluids. The radiation pressure at a rigid interface is obtained in the limit when the impedance of one of the fluids becomes infinite.

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## FIGURE CAPTIONS

**Figure 1** Apparatus to measure the force between two rigid parallel plates due to the radiation pressure of broadband acoustic noise. The tank is made of 0.64 cm steel, and has length 1.5 m and diameter 0.5 m. One end is ellipsoidal while the other is flat with a steel faceplate to which is mounted a 5 cm thick, 61.5 cm square acrylic access cover secured by four C-clamps. A sliding 5 cm thick acrylic bar allows the positioning of the balance at a desired location, and also serves for spectral measurements of the noise along the tank. A microphone was positioned within 1 cm of the top plate to provide spectrum intensity measurements.

**Figure 2** Experimental spectrum in a band of frequencies between 4.8 – 16 kHz. The spectrum is relatively flat and exhibits structure (dip) and an overall 5 dB roll off. The total intensity of the noise is 133 dB (re $10^{-12}$ W/m$^2$).

**Figure 3** Force between two parallel rigid plates as a function of the distance between them. The points are experimental data, and the curves are form theory (2) and (3) with no adjustable parameters for a flat spectrum (solid curve) and a piecewise power–law spectrum that approximates the experimental spectrum (dashed curve).

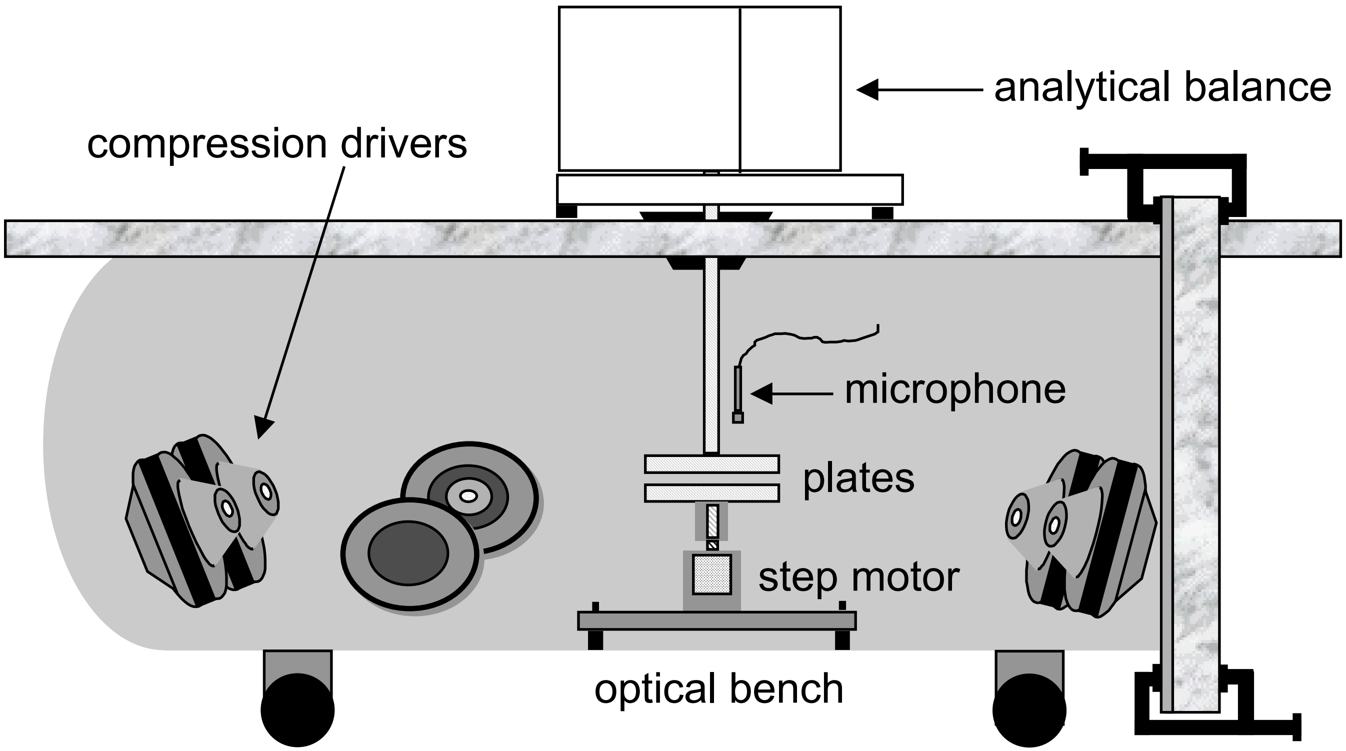
analytical balance
compression drivers
microphone
plates
step motor
optical bench

**FIGURE 1**

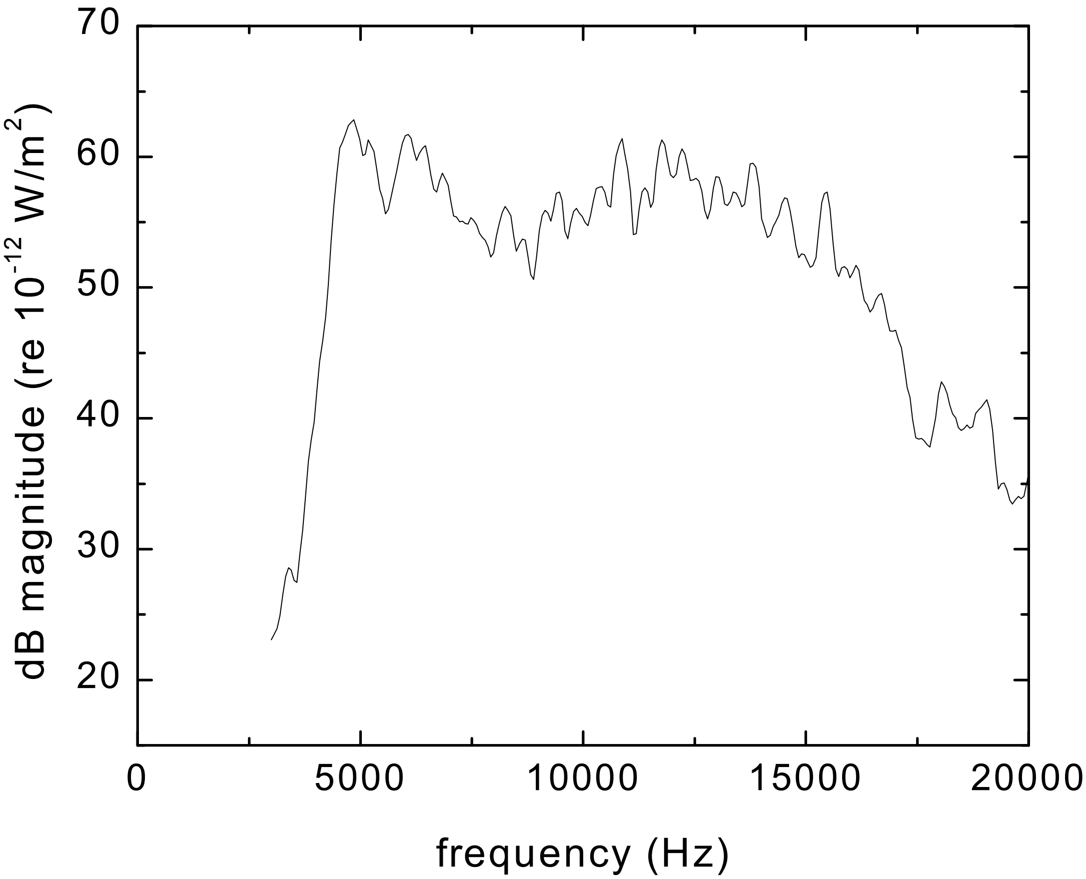
70
60
50
40
30
20
dB magnitude (re 10^-12 W/m^2)
0
5000
10000
15000
20000
frequency (Hz)

**FIGURE 2**

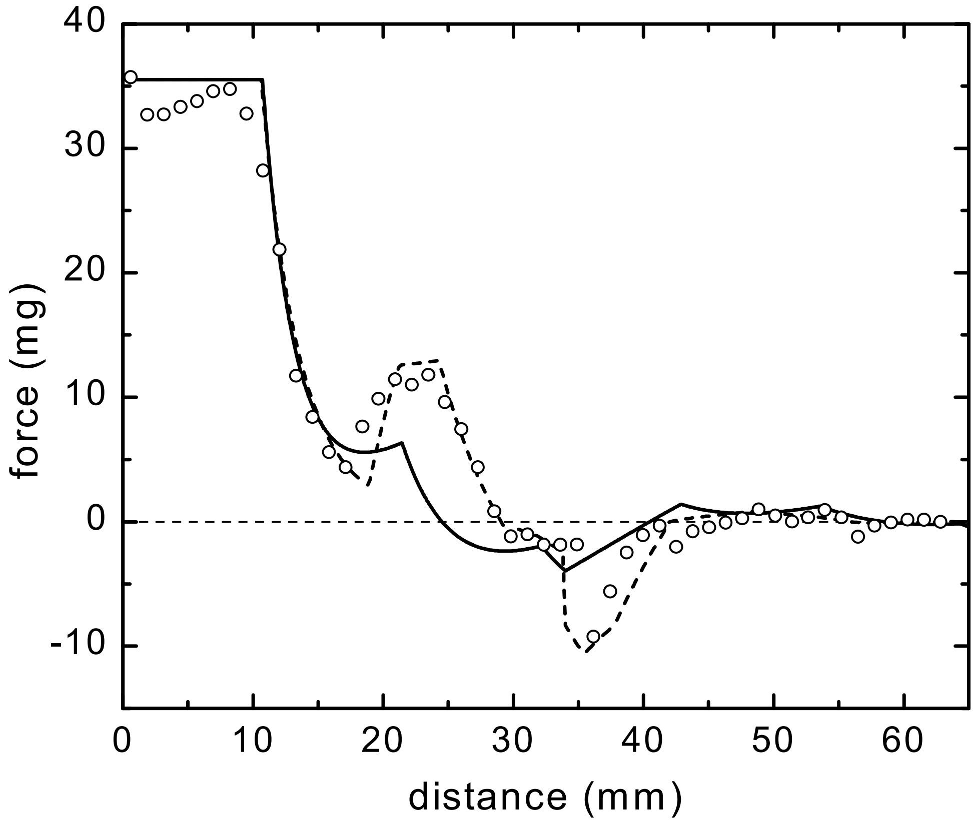


**FIGURE 3**